Papoulis-Gerchberg Hybrid Filter Bank Receiver for Cognitive-/Software-Defined Radio Systems

José P. Magalhães, Teófilo Monteiro, and José M. N. Vieira IEETA, Universidade de Aveiro Aveiro, PORTUGAL Roberto Gómez-García Dpt. Signal Theory and Communications University of Alcalá Alcalá de Henares, SPAIN Nuno B. Carvalho Instituto de Telecomunicações Universidade de Aveiro Aveiro, PORTUGAL

Abstract-Emerging Software-Defined Radios (SDRs) should be prepared to deal with wide-band sparse-spectrum RF signals. This requires the availability of advanced analog-to-digital frontends with a fast sampling rate, large dynamic range and capable of handling high peak-to-average-power-ratio (PAPR) signals. A Hybrid Filter Bank (HFB) receiver solution was recently proposed by the authors to address such demanding requirements in the SDR context, with special emphasis on the RF analog part. In this paper, as further research, a maximally-decimated fivechannel HFB for its intermediate-frequency (IF) part is shown. Unlike its previously reported RF counterpart, it makes use of the Papoulis-Gerchberg algorithm to attain a more efficient implementation in terms of computational cost and reconfigurability. For this HFB-based IF SDR receiver, the digital filters compensating the analog design imperfections are evaluated and realtime signal-reconstruction tests for wide-band and narrow-band signals are carried out. A sensitivity discussion is also provided.

I. INTRODUCTION

Software-Defined Radios (SDRs) and Cognitive Radios (CRs) have become very promising solutions to support modern wireless communications applications [1]. Spectral dynamic access, frequency recognition, inter-operability and reconfigurability are some of the main features which have made SDRs and CRs top priorities either for military or civilian purposes [2]. However, several challenges must be first faced up before having a fully-operative SDR system with these characteristics. For example, its receiver block must be capable of sensing ultra-broad-band spectrums while dealing with very high and low power signals at the same time. This demand can hardly be attained with state-of-the-art analogto-digital converters (ADCs) owing to the irreconcilability between sampling speed and resolution [3]. Even in the future, such requisites may never be achieved due to the slow evolution of the ADC technology in relation to the More's Law [4].

The aforementioned limitations can be circumvented through a receiver approach inspired on a "Hybrid Filter Bank" (HFB), where the input signal is frequency multiplexed into several bandpass sub-bands that are subsequently converted to the digital domain through sub-sampling [5]. Thus, this parallel structure alleviates the sampling-rate requirements for the ADCs as signal components with narrower bandwidths must be handled by them (see Fig. 1). Consequently, improved trade-offs in terms of conversion resolution become feasible and, therefore, the receiver dynamic range is increased. The HFB is completed by means of a digital synthesis stage, where digital filters are utilized to eliminate the aliasing components coming from the sub-sampling and the amplitude and phase distortions inherent to the real analog frequency multiplexer. Several methods to properly evaluate the synthesis digital filters have been presented. Moreover, in [6] and putting attention to the inversion of the RF part, it was shown by the authors that a nearly perfect reconstruction is possible with this alternative. This means that the overall HFB receiver behaves just like a single ADC with improved performance and an extra time delay attributable to the digital filtering action.

In this work, as further research, a maximally-decimated five-channel HFB for an intermediate-frequency (IF) SDR receiver is described. Unlike its RF counterpart of [6], it exploits the Papoulis-Gerchberg algorithm to improve computational cost and system reconfiguration. A proof-on-concept IF SDR receiver prototype is developed and reconstruction experiments for narrow-band and wide-band signals exhibiting 16-level quadrature amplitude modulation (QAM) are performed.

II. HFB DESIGN METHODOLOGY

Fig. 1 shows the conceptual structure of the conventional M-channel maximally-decimated HFB to be employed in the SDR IF receiver. If a limited bandwidth B is assumed for the input signal x(t) after being filtered by an anti-aliasing filter, then the bandpass sampling formalism can be applied to it [7]. In this case, under sub-sampling, the required sampling rate for the HFB-based receiver is $f_s = 2B/M$, which is M times lower than in a traditional direct-sampling receiver. This sampling-rate reduction enables better trade-offs to be attained in terms of conversion resolutions which will result on an increased receiver dynamic range. However, the analog frequency channelizer splitting the input signal into different frequency sub-bands irremediably causes phase and magnitude distortion in each channel, as non-ideal filters are used. Also, aliasing terms may appear owing to both the sub-sampling process and the finite selectivity of the analog filters. Nevertheless, these undesired effects can be compensated at the digital level through a properly designed digital synthesis filtering stage. The method adopted here for the evaluation of this synthesis block, carefully described in [8], is summarized below.



Fig. 1. Conceptual structure of the M-channel maximally-decimated HFB ("AFB" and "SFB" denote "analysis filter bank" and "synthesis filter bank").

The synthesis filters and the analog filters are related through the following equation concerning the whole system:

$$T_p\left(e^{j\omega}\right) = \frac{1}{MT_s} \sum_{m=0}^{M-1} H_m\left(j\frac{\omega}{T_s} - j\frac{2\pi p}{MT_s}\right) F_m\left(e^{j\omega}\right),$$
$$p = 0, 1 \dots M - 1 \tag{1}$$

where T_s is the sampling period, H_m represents the frequency response of the analog filters, F_m is the frequency response of the digital synthesis filters, $T_{p=0}$ refers to the complete frequency response of the HFB in terms of distortion, and $T_{p\neq0}$ are the aliasing components of the overall system. For a perfect signal reconstruction, the following must be met:

$$T_p\left(e^{j\omega}\right) = \begin{cases} e^{-j\omega d} & \text{for } p = 0\\ 0 & \text{for } p = 1, 2 \dots M - 1, \end{cases}$$
(2)

which means that the desired distortion for the complete HFB is merely a time delay d whereas the aliasing terms are zero.

By comparing (1) and (2), it is deduced that F_m is obtained through the inversion of H_m in relation to the intended T_p . Moreover, if a finite impulse response (FIR) implementation is chosen, the evaluation of the coefficients for the digital filters is carried out through simple Fourier transformations. Unfortunately, due to the use of FIR filters, a perfect reconstruction as defined by (2) is in general not possible owing to the needed excessive number of coefficients [8]. Thus, a compromise between the order of the filters (i.e., length of the impulse responses) and the performances of the resulting HFB with regard to aliasing and distortion cancelation must be taken.

III. APPLICATION OF THE PAPOULIS-GERCHBERG Algorithm to the HFB Design

With the aim of reducing the implementation cost of the digital filters without sacrificing the system performances for certain frequency bands, the Papoulis-Gerchberg algorithm can be used in the HFB design [9]. In the framework of the HFB theory, it allows to optimize the digital synthesis filter responses in some specific spectral regions, called "bands of interest". Indeed, through this algorithm, the digital filter responses are iteratively re-evaluated and truncated in time domain to assure low distortion and aliasing levels for these frequency bands but not outside them. Thus, it is possible to reduce the order of the filters, making them suitable for real-time realizations. The application of the Papoulis-Gerchberg technique to the HFB design is carried out through the following steps:

- 1) Evaluate the synthesis filters for all the spectral axis ω using a certain number of coefficients L.
- 2) Define ω_I and ω_N as the spectral regions of interest and of no interest, respectively, so that $\omega_I \bigcup \omega_N = \omega$.
- 3) Calculate the HFB transfer function through (1). This will result in $\tilde{T}_p(e^{j\omega})$ which, owing to the finite-length restriction for the synthesis filter impulse responses, will have non-negligible terms of aliasing and distortion.
- 4) From the previous step, ensure that

$$\widetilde{T}_{p}\left(e^{j\omega}\right) = \begin{cases} T_{p}\left(e^{j\omega}\right) & \text{for } \omega = \omega_{I} \\ \widetilde{T}_{p}\left(e^{j\omega}\right) & \text{for } \omega = \omega_{N}, \end{cases}$$
(3)

which means imposing the perfect reconstruction conditions of (2) for the bands of interest ω_I . In the remaining spectral regions $\omega = \omega_N$, the distortion and aliasing terms coming from the previous evaluation are kept.

- 5) Re-evaluate the synthesis digital filters through the HFB methodology here presented but now using $\tilde{T}_p(e^{j\omega})$ as the new goal for the desired HFB frequency response.
- 6) Truncate the resulting synthesis filter impulse response $f_m(n)$ to the new desired length $\widetilde{L} < L$, as follows:

$$\widetilde{f}_m(n) = f_m(n)w(n), \quad m = 0, 1...M - 1, \quad (4)$$

which obtains $\tilde{f}_m(n)$ through the multiplication of $f_m(n)$ by a "box function", w(n), defined as

$$w(n) = \begin{cases} 1 & \text{for } 0 < d - |\widetilde{L}/2| < n < d + |\widetilde{L}/2| < L \\ 0 & \text{otherwise} \end{cases}$$
(5)

where d is the time delay assumed for the HFB response. 7) Go back to 3) using now $\tilde{f}_m(n)$ to compute $F_m(e^{j\omega})$. Note that, in addition to a lower implementation cost, the Papoulis-Gerchberg algorithm leads to considerable DC power savings for the HFB-based receiver system. Indeed, in a real CR/SDR scenario, it is not usual for the signal of interest to cover the entire bandwidth of the receiver. Thus, by means of the HFB approach, only those channels inside and closer to the frequencies of interest have to be turned on to process it, resulting in most of the branches powered off at that moment.

IV. EXAMPLE OF HFB-BASED IF SDR RECEIVER AND REAL-TIME SIGNAL-RECONSTRUCTION TESTS

Following the previous approach, a proof-of-concept HFBbased IF SDR receiver prototype has been implemented and signal-reconstruction experiments have been conducted. The designed receiver system is a five-channel HFB which operates from 50 MHz up to 75 MHz (i.e., 25 MHz of bandwidth). The schematic of the experimental procedure for the signalreconstruction tests is given in Fig. 2. The goal is to evaluate the performances of the devised HFB receiver with regard to an equivalent single-ADC bandpass-sampling receiver. This



Fig. 2. Schematic of the testing procedure for the real-time signalreconstruction experiment.



Fig. 3. Channel transfer functions of the employed analog filter bank (five lower channels of the lumped-element inverted cochlea IF channelizer of [10]).

means comparing the reconstructed signal $\hat{x}[n]$ with the original signal x[n] as acquired by the conventional receiver. For the HFB-based receiver, the sampling rate in each ADC is 10 Msps, that is, five times lower than the minimum one imposed by the Nyquist theorem and adopted for the directsampling case. To generate the real-time signals, a SMU 200A vector signal generator from Rohde & Schwarz was utilized. A Virtex-6 FPGA with an FMC108 4DSP 8-ADC board of 14 bits of resolution was employed for the signal acquisition. To realize the analog filter bank, the five lower channels of the eight-channel lumped-element IF channelizer developed in [10] and bio-inspired in an inverted cochlea solution were employed, by loading the unused ports with 50- Ω impedances. Fig. 3 draws their channel transfer functions, measured with a HP-8720C vectorial network analyzer (VNA) from Agilent.

For the real-time experiment, two different tests were carried out below. In the first one, the HFB-based IF SDR receiver is feeded with a wide-band signal so that the synthesis filters are evaluated in the whole radio bandwidth. After that, a narrowband signal is chosen and the Papoulis-Gerchberg algorithm is applied to optimize the synthesis filters only for the band of interest. In both cases, the signal reconstruction is done and the approximation errors relative to the time-delayed signal as processed by the direct-sampling receiver are detailed.

A. Wide-Band Signal Reconstruction

For the acquisition of the wide-band signal, the synthesis filters of the HFB must be first evaluated for the entire radio bandwidth. Therefore, using the methodology expounded in Section II, five digital filters with only 200 coefficients have been derived. The resulting channel frequency responses are



Fig. 4. Frequency response magnitude of the evaluated synthesis digital filters for the wide-band signal experiment (L = 200).



Fig. 5. Estimated distortion and aliasing levels in the HFB for the wide-band signal experiment (L = 200).

shown in Fig. 4. Note that that these filters were designed to operate at baseband frequencies as the bandpass sampling process will downconvert the original spectrum from the third to first Nyquist Zones. Thus, having projected the complete HFB, its performances may be estimated through the expression (1). For the selected number of coefficients, an approximate value of -40 dB of distortion and aliasing terms are predicted in the reconstructed signal $\hat{x}[n]$ in comparison with x[n] (see Fig. 5).

For the real-time reconstruction experiment, a 16-QAM test signal with a rate of 20 Msymb/s and a carrier frequency of 62.5 MHz was chosen. The reconstructed and original signals are compared in Fig. 6 in frequency and time domain, showing a fairly-close agreement for both cases. In the time domain, where the representation was normalized to the maximum of x[n], the computed deviation mean error was ≈ 0.05 .

B. Narrow-Band Signal Reconstruction

In the second test, a 16-QAM signal with a bit rate of 1 Msymb/s and a carrier frequency of 68 MHz was generated. The Papoulis-Gerchberg algorithm, as detailed in Section III, is then applied to the previous HFB evaluation to optimize its response only for the band of interest (65 MHz to 71 MHz). The new synthesis filter responses for L = 200 coefficients are plotted in Fig. 7. As shown, only those filters near or inside the band of interest are considered in the reconstruction process.

Fig. 8 plots the attained reconstruction results in frequency and time domain. In this case, the reconstruction error is even lower, about 0.025 between the normalized $\hat{x}[n]$ and x[n].

Here, it could be checked through the power spectrum analysis of the error signal that although the reconstruction



Fig. 6. Frequency and time domain (normalized to max $\{|x[n]|\}$) responses of the reconstructed $(\hat{x}[n])$ and original (x[n]) signals and approximation error for the wide-band signal experiment $(e[n] = |\hat{x}[n] - x[n]|)$.



Fig. 7. Frequency response magnitude of the evaluated synthesis digital filters for the narrow-band signal experiment (L = 200).

results for the narrow-band signal were in conformity with its predictions, some degradation arises for the wide-band signal test in relation to the estimated error from Fig. 5. This is due to the fact that the parallel structure is very sensitive to analog circuitry imperfections that could appear, leading to an inaccurate H_m representation of the analog system behavior [11]; for example, the VNA measurement uncertainty, temperature drifts or electromagnetic interference during the experiments. However, the most critical problem in this case is the phase imbalance between the ADCs of the selected DSP board, that could be of three degrees for the considered frequency range.¹ This phenomenon will unavoidably cause the signals after the synthesis filters to be summed out of phase, so that the distortion and aliasing terms are not canceled as expected. Techniques to counteract this effect are currently under study.

V. CONCLUSION

An IF SDR receiver architecture based on the HFB formalism has been described. To design the synthesis digital filter bank for the analog inversion, the Papoulis-Gerchberg algorithm has been introduced, leading to benefits in terms of computational cost, DC power saving and system reconfigurability. A proof-on-concept five-channel IF SDR receiver circuit using this approach has been built, and signal-reconstruction tests for this type of receiver have been performed for the first time. Specially for the narrow-band signal, excellent reconstruction results have been obtained. On-going research work is the

¹This is not a problem for the narrow-band signal, where the ADCs inequalities become a minor effect as it is mostly contained in a single channel.



Fig. 8. Frequency and time domain (normalized to max $\{|x[n]|\}$) responses of the reconstructed $(\hat{x}[n])$ and original (x[n]) signals and approximation error for the narrow-band signal experiment $(e[n] = |\hat{x}[n] - x[n]|)$.

improvement of the system sensitivity robustness to undesired effects such as the FPGA ADCs inequalities, through an embedded network analyzer or adaptive filtering techniques, measuring the system response —from the anti-aliasing filter to the ADCs— for a complete control of the compensation.

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